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Thermal conductivity measurement of mercury in a magnetic field

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Abstract—The thermal conductivity of mercury is measured in a magnetic field of between 0 and 4 T at 269 K by the transient hot-wire method with a ceramic probe. Suppression of the convection caused by heating the wire is experimentally confirmed. Complete suppression of convection is achieved above about 1 in $\log_{10}(Ha/Ra^{1/3})$.

1. INTRODUCTION

Two TYPES of convection take place during the thermal conductivity measurement of liquids with the transient hot-wire method; one results from inhomogeneity of the temperature distribution in the liquid and the other from the heating of the liquid around the probe wire. If perfect isothermal conditions were achieved around the liquid, the former convection would not take place. On the other hand, it is rather more difficult to suppress the occurrence of the latter type convection since the liquid heated by the wire will rise spontaneously, particularly when a long thin wire is used as a probe. As is well known in measurements by the transient hot-wire method [1], temperature increase in thin wires is proportional to logarithmic time in the early stages of measurements; however, it will begin to deviate from a linear relationship as time passes if an amount of heat sufficient to cause convection is applied to the liquid around the wire. Therefore, measurement may be performed only to the point at which convection takes place. In order to increase the time range over which measurement may be performed, it becomes necessary, then, to suppress or postpone the occurrence of convection, but this has been extremely difficult to achieve, and no notable success in the area has been reported to date.

Two approaches may be considered for suppressing convection. One is utilization of a microgravity environment [2, 3], since convection is essentially due to the buoyancy force in liquids. If thermal conductivity measurements could be performed under microgravity conditions, the above two types of convection, both buoyancy related, would be sufficiently suppressed, and it would be possible to determine the correct thermal conductivity.

The other possible approach to suppression is utilization of a magnetic field, but this would be effective only for electromagnetic fluids. It is well known that the onset of convection is delayed in an external magnetic field [4].

In order to study the effect of a high magnetic field on buoyancy convection in a measurement cell, we measured the thermal conductivity of mercury under a magnetic field of between 0 and 4 T, at 269 K, using a previously developed ceramic probe [5, 6]. Since this measurement was performed at room temperature, there was no apparent pre-measurement convection stemming from imhomogeneous temperature distribution. This experiment, then, could be used to determine the effect of an external magnetic field on the induction of convection by wire heating.

2. EXPERIMENTS

The measurements were performed with a ceramic probe using the transient hot-wire method. A printed wire on a thick alumina substrate (rather than the standard thin, bare, metallic wire) was used to measure the temperature increase occurring when a constant electric power was input to that wire [5]. Takegoshi *et al.* have given a working equation, based on the assumption that the wire is an ideal line heat source on a flat interface between two media [7]

$$\lambda_L + \lambda_S = \frac{Q}{2\pi} \Big/ \frac{d(\Delta T)}{d(\ln(t))} \tag{1}$$

where ΔT is the temperature increase of the wire, and λ_L and λ_S the thermal conductivities of the liquid sample and alumina substrate, respectively. Q is the constant input power to the wire per unit length; this

NOMENCLATURE

- B external magnetic field [W m^{-2}]
- g gravitational acceleration, 9.81 m s^{-2}
- I current applied to wire [A]
- l distance between hot wire and wall of crucible [m]
- L length of wire [m]
- Q constant input power per unit length of wire, IR_w^2/L [W m^{-1}]
- R_w electrical resistance of wire [Ω]
- t time [s]
- ΔT temperature increase of wire [K]
- ΔT_{av} averaged temperature increase of wire [K].

Greek symbols

- α thermal diffusivity [m^2 s^{-1}]
- β volumetric coefficient of thermal expansion [K^{-1}]
- λ_C contribution of convection on apparent thermal conductivity [W m^{-1} K^{-1}]
- λ_L thermal conductivity of liquid (mercury) [W m^{-1} K^{-1}]
- λ_S thermal conductivity of substrate [W m^{-1} K^{-1}]
- ν kinematic viscosity [m^2 s^{-1}]
- ρ density [kg m^{-3}]
- σ electrical conductivity [Ω^{-1} m^{-1}]
- χ average value of temperature coefficient of thermal conductivities [K^{-1}].

Dimensionless numbers

- Ha Hartman number, $(\sigma/\nu\rho)^{1/2}Bl$
- Nu Nusselt number, $(\lambda_L+\lambda_C)/\lambda_L$
- Nu(Re) reconstructed Nusselt number, see equation (2)
- $Nu(Ha = 0)$ Nusselt number without external magnetic field
- Ra Rayleigh number, $g\beta\Delta T_{av}l^3/\nu\alpha$.

was determined by I^2R_w/L, where I is current and R_w the resistance of the wire during measurement. L is the length of the wire. R_w was 6.06 Ω at 269 K.

Figure 1 shows the structure of a measurement cell contained within a superconducting magnet. A ceramic probe ① is set within a carbon crucible ②, enclosed in an aluminum cartridge ③. The rod shaped ceramic probe was made in a process similar to one which was reported previously [5]. A thick substrate was first formed by laminating alumina green sheets. A platinum wire and electrodes were printed on the thick substrate. The wire was 15 μm thick, 100 μm wide and 70 mm long. The surface on which the wire and electrodes were printed was coated with a 60 μm alumina insulation layer. The probe was then co-fired at about 2000 K and machined into a rod shape. With this processing, the radial distance from the wire to the outside of the alumina substrate became 14 mm. The length of the printed wire between the potential electrodes L was 70 mm. Platinum lead wires of 0.8 mm diameter were connected to the printed electrodes at the end of the probe. The space between the carbon crucible and the probe was filled with mercury.

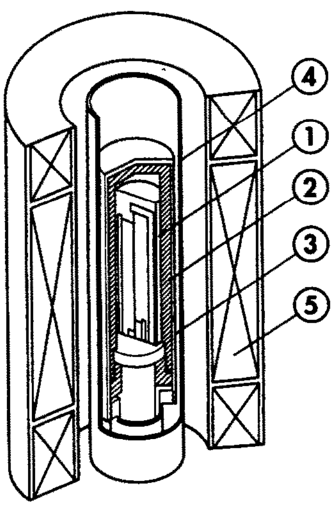

FIG. 1. Structure of measurement cell within a superconducting magnet: ①, ceramic probe; ②, carbon crucible (hatched); ③, aluminum cartridge; ④, stainless steel inner tube; ⑤, superconductor magnet.

The superconducting magnet ⑤ was 50 mm in diameter and 350 mm long. It produced a magnetic field in the vertical direction in an inner tube ④. The strength of the magnetic field could be varied up to a maximum of 8 T. Along the cartridge in the longitudinal direction, magnetic strength varied by about 1.5%. The atmosphere in the inner tube was air.

The printed wire was firmly fixed under the insulation layer on a co-fired alumina substrate. Current flow through the wire and electrodes was parallel to the magnetic flux. Thus the printed wire and electrodes were not damaged by Lorentz force.

The measurement system consisted of a constant current supply and an A/D converter [8]. When the constant current source had supplied power to the printed wire, its resistance was directly measured and converted by the A/D converter. Measurement started from 1.1 s after application of the current. The sampling rate was 10 times per second.

The uncertainty of Q was 1.6% which contained the uncertainty of wire length, wire resistance, and current. The uncertainty of $d(\Delta T)/d(\ln(t))$ was about 1.5% which was estimated by deviations from linearity in $\Delta T - \ln(t)$ curves. The overall accuracy for this measurement system was estimated to be better than ±3.5%.

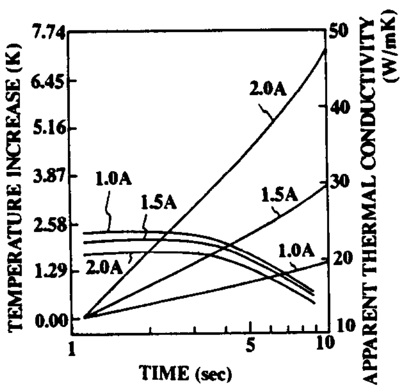

FIG. 2. Temperature increase and apparent thermal conductivity vs ln (t) for alumina substrate of ceramic probe.

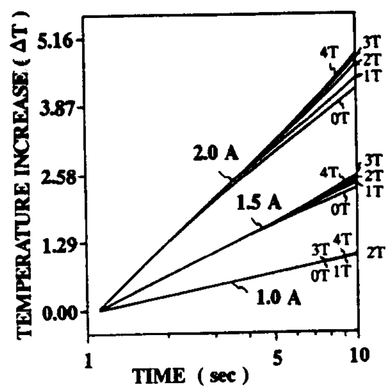

FIG. 3. Temperature increase vs ln (t) for mercury. Magnetic field varied from 0 to 4 T.

3. RESULTS AND DISCUSSION

3.1. *Thermal conductivity of the alumina substrate*

The thermal conductivity of the alumina substrate was measured at 273 K in an air atmosphere. Since the thermal conductivity of alumina is three orders of magnitude greater than that of air, heat transfer due to air convection may be considered negligible at room temperature. Therefore, the slope of the ΔT curve with respect to ln (t) in equation (1) is inversely proportional to the thermal conductivity of the alumina substrate alone.

Figure 2 shows ΔT as a function of ln (t) for three cases of input current: 1.0, 1.5, and 2.0 A. Input powers for these currents correspond to about 87.6, 201, and 369 W m^{-1}, respectively. Apparent thermal conductivities, also shown in Fig. 2, were calculated using equation (1). The values of the apparent thermal conductivities were mostly constant, which indicates that the ΔT curves were linear before about 4 s. The ΔT curves deviated from the linear relationship after about 4 s. This deviation was due to the fact that the heat from the wire reached the outside of the probe. Thus measurements with this ceramic probe were significant up until about 4 s. Hayashi *et al.* have experimentally determined the critical time T_d after which the deviation of ΔT from a straight line begins [9]. T_d calculated with their equation was 3.48 s for the present probe.

The apparent thermal conductivities were current dependent. This may mainly be attributed to variations in thermal conductivity due to changing temperatures in alumina. The apparent thermal conductivities at 3 s were 23.6 W m^{-1} K^{-1} for the case of 1.0 A, 22.3 W m^{-1} K^{-1} for 1.5 A and 20.6 W m^{-1} K^{-1} for 2.0 A. Reference temperatures were calculated according to de Groot *et al.* [1]. The reference temperatures were 279.4, 288.8, and 301.9 K. The thermal conductivities, estimated using a previously reported empirical equation [6] were 24.1, 23.2, and 22.0 W m^{-1} K^{-1}, respectively at the reference temperatures. The difference between the two thermal conductivities at reference temperatures increased with increasing temperature. This may be due to an undetermined source within the present measurement system. However, the thermal conductivity extrapolated from these experimental data was about 24.5 W m^{-1} K^{-1} at 273 K. This was in good agreement with the 24.7 W m^{-1} K^{-1} estimated from the empirical equation.

3.2. *Thermal conductivity of mercury in a magnetic field*

The thermal conductivity of mercury was measured at 269 K. Figure 3 shows ΔT vs ln (t) for currents of 1.0, 1.5, and 2.0 A, in various magnetic flux densities from 0 to 4 T. Figure 4 shows the sums of apparent conductivities calculated according to equation (1). For input currents of 1.5 and 2.0 A, convection took place with wire heating where no magnetic field was applied, for we may observe that their apparent thermal conductivities in Fig. 4 increase as time elapses.

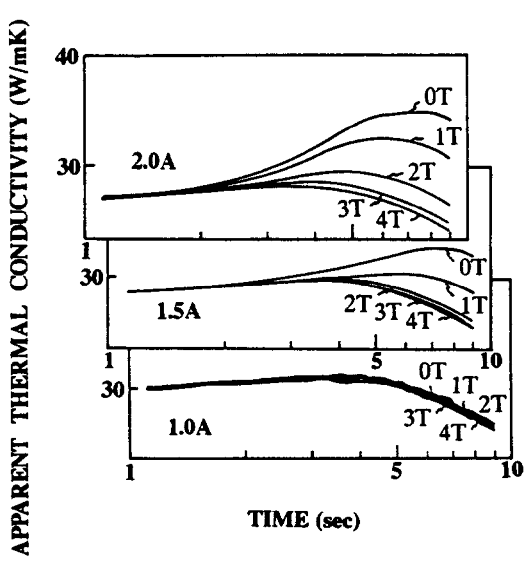

FIG. 4. Apparent thermal conductivity vs ln (t) for mercury. Magnetic field varied from 0 to 4 T.

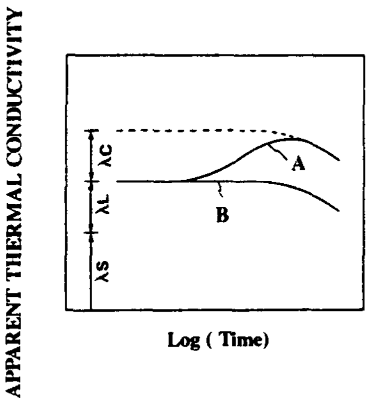

FIG. 5. Determination of λ_C. λ_C is contribution of convection in steady-state flow.

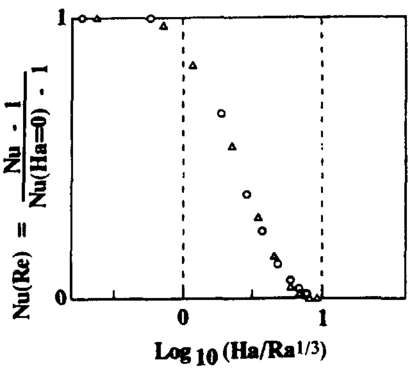

FIG. 6. Reconstructed Nusselt numbers vs $\log_{10}$ $(Ha/Ra^{1/3})$ for input currents of 1.5 A (△) and 2.0 A (○).

The convection by wire heating decreased with increasing magnetic flux density. The effect seen in Fig. 3, in which the ΔT curves became linear with increasing magnetic field for currents of 1.5 and 2.0 A, is more clearly seen in Fig. 4.

On the other hand, the apparent thermal conductivities measured with a current of 1.0 A were little influenced by magnetic fields, as shown in Fig. 4. The apparent thermal conductivities for a current of 1.0 A were mostly constant before 4 s, after which they decreased, as shown in Fig. 4. The shape of those curves of apparent thermal conductivity is similar to that in Fig. 2 for the apparent thermal conductivities of alumina substrates. Mercury may be regarded as solid in the 1.0 A measurement. All of this suggests that wire heating at 1.0 A did not produce convection. Even if the convection had taken place, it would have been too weak to be detected by $\Delta T - \ln(t)$ curves. The roughness of the line of apparent conductivity for 1.0 A was mainly due to limits in the degree of resolution of the measurement system, since the resistance increase directly measured by the A/D converter was much smaller than the initial resistance of the wire. The total increase from the initial resistance for a current of 1.0 A was about 0.3%.

The thermal conductivity was determined by subtracting the apparent thermal conductivity of the alumina substrate from the thermal conductivity sum. The range of possible measurement was limited to less than 4 s by the diameter of the probe, as has been previously noted. Temperature increases before 2 s were excluded because of the thermal resistance between the probe and the mercury. Therefore, the thermal conductivity sum was that obtained at 3 s at 4 T. The thermal conductivity for the alumina substrate was that taken from previously obtained data [6]. The extrapolated thermal conductivity was 6.7 W m^{-1} K^{-1} at 269 K, which was 13% smaller than the 7.7 W m^{-1} K^{-1} of ref. [10].

Figure 5 shows apparent conductivity vs $\ln(t)$ as measured with convection (line A) and without convection (line B). On the assumption that convection becomes steady state in late stages of the measurement period. contribution of steady-state convection to apparent thermal conductivity λ_C may be defined by the dashed line shown in Fig. 5. We defined a Nusselt number Nu $(= (\lambda_L + \lambda_C)/\lambda_L)$. Thus, λ_C and Nu can be determined if the dashed line and line B (without convection) have the same shape. Line B was determined from the apparent conductivities for 1.0 A in Fig. 4. Figure 6 shows a reconstructed Nusselt number $Nu(\mathrm{Re})$ as a function of $\log_{10}$ $(Ha/Ra^{1/3})$. The reconstructed Nusselt number may be defined as

$$Nu(\mathrm{Re}) = \frac{Nu - 1}{Nu(Ha = 0) - 1} \tag{2}$$

where $Nu(Ha = 0)$ is an average Nusselt number obtained under conditions where there is no magnetic field [11]. Ha and Ra are Hartman and Rayleigh numbers. We used the same length l as the characteristic length for both Ha and Ra. Average temperature increases ΔT_{av} during the measurement were used in Ra. ΔT_{av} for 1.5 and 2.0 A were 10.8 and 20.2 K. Experimental data of 1.5 and 2.0 A were consistently plotted against $\log_{10}$ $(Ha/Ra^{1/3})$, Fig. 6. This result was similar to that obtained by Ozoe and Maruo [11]. Figure 6 suggests that complete suppression of convection might be achieved above about $\log_{10}$ $(Ha/Ra^{1/3}) = 1$.

Heated mercury rises vertically. Since the magnetic field is also vertical, no Lorentz force would apply to this motion. However, there exist horizontal motions at the upper and lower areas of the mercury. This motion might be suppressed by a Lorentz force, and the suppression of horizontal motion might result in the suppression of vertical motion.

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MESURE DE LA CONDUCTIVITE THERMIQUE DU MERCURE DANS UN CHAMP MAGNETIQUE

Résumé—La conductivité thermique du mercure est mesurée dans un champ magnétique de 0 à 4 T, à 269 K, par une méthode de fil chaud transitoire, avec une sonde en céramique. La suppression de la convection causée par le chauffage du fil est expérimentalement confirmée. La suppression complète de la convection est atteinte lorsque $\log_{10}(Ha/Ra^{1/3})$ est à peu près égal à 1.

MESSUNG DER WÄRMELEITFÄHIGKEIT VON QUECKSILBER IN EINEM MAGNETISCHEN FELD

Zusammenfassung—Mit Hilfe des instationären Heißdrahtverfahrens und einem keramischen Sensor wird die Wärmeleitfähigkeit von Quecksilber in einem magnetischen Feld zwischen 0 und 4 T bei einer Temperatur von 269 K gemessen. Die Unterdrückung der thermischen Konvektion wird im Versuch nachgewiesen. Eine vollständige Unterdrückung ergibt sich für $\log_{10}(Ha/Ra^{1/3}) > 1$.

ИЗМЕРЕНИЕ ТЕПЛОПРОВОДНОСТИ РТУТИ В МАГНИТНОМ ПОЛЕ

Аннотация—Нестационарным тепловым методом проведены измерения теплопроводности ртути в магнитном поле в диапазоне интенсивности от 0 до 4 Т при температуре 269 К с использованием керамического зонда. Экспериментально подтверждается подавление конвекции, вызванной нагревом проволоки. Полное подавление конвекции достигается при значениях свыше 1 в $\log_{10}(Ha/Ra^{1/3})$.